Recitation 1. February 23

Focus: Rules of matrix multiplication, Gaussian and Gauss-Jordan elimination.

The most basic rule that you should remember: **row column**. It shows the order in which you write or compute:

- The first index denotes the row, the second number the column.
- You multiply a row by a column to get a number.
- An $m \times n$ matrix has m rows and n columns.

The formula **left matrix multiplication corresponds to row operations** explains the mathemagic behind Gaussian or Gauss-Jordan elimination. More precisely, performing row operations on a matrix A is the same as doing LA for some other matrix L, which turns out to be a products of elimination, diagonal and permutation matrices.

1. *Rules of matrix multiplication. (Section 2.4 of Strang.)* Consider the following matrices:

$$A = \begin{bmatrix} 1 & 2 & 0 \\ 3 & -1 & 2 \end{bmatrix}, \qquad B = \begin{bmatrix} -1 & 0 & 1 \\ -1 & 3 & 2 \end{bmatrix}, \qquad C = \begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix}, \qquad D = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$$

Which of these matrix operations are allowed?

a) AB
b) $(A+B)C$
c) $C(A+B)$
d) AD
e) DA
f) CAD

2. *Binomial formula for matrices.* Show that $(A+B)^2$ is different from $A^2 + 2AB + B^2$ when

$$A = \begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix}, \qquad B = \begin{bmatrix} 1 & 0 \\ 3 & 0 \end{bmatrix}.$$

Write down the correct rule: $(A+B)^2 = A^2 + \cdots + B^2$.

Solution:

3. Consider the following matrices:

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}, \qquad B = \begin{bmatrix} 2 & 0 \\ 0 & -1 \end{bmatrix}, \qquad C = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \qquad D = \begin{bmatrix} 0 & 1 \\ 0 & -1 \end{bmatrix}$$

How is each row of BA, CA, DA related to the rows of A?

Solution:

4. *Gaussian elimination (row echelon form).* Solve the following system of linear equations by Gaussian elimination:

$$\begin{cases} x + 2y + 3z = 1 \\ \quad\;\; y + \;\; z = 2 \\ 3x + \;\; y - \;\; z = 3 \end{cases}$$

Solution:

5. *Gauss-Jordan elimination (reduced row echelon form).* Same problem as above, but do Gauss-Jordan elimination.

Solution: